# ANALYSIS III MIDTERM EXAMINATION

Total marks: 40
Attempt all questions
Time: 2 hours (2 pm - 4 pm)

(1) Let $S$ be the set of points $(x, y, u, w)$ in $\mathbb{R}^4$ satisfying the equations $x^2 + y^2 + u^2 - 3v = 0$ and $2x + xy - y + 3u^2 - 9v = 0$. Find all points of $S$ for which there is a neighborhood in which $S$ is a smooth 2-surface. State the theorem you are using to do this problem. (5+5 = 10 marks)

(2) Briefly explain the Lagrange multiplier method (without proofs). Use the Lagrange multiplier method to find the maximal and minimal values of $f(x, y, z) = x - 2y + 3z$ on the sphere $x^2 + y^2 + z^2 = 1$. (5+5 = 10 marks)

(3) Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Show that the graph $G(f) := \{(x, y) \in \mathbb{R}^2 | y = f(x)\}$ of $f$ has measure zero in $\mathbb{R}^2$. Hint: Use the fact that $f$ is uniformly continuous. (10 marks)

(4) Let $U \subset \mathbb{R}^2$ be an open subset and let $f : U \to \mathbb{R}$ be a $C^2$ function. Use Fubini's theorem and the fundamental theorem of calculus to show that $\frac{\partial^2 f}{\partial x \partial y}(a, b) = \frac{\partial^2 f}{\partial y \partial x}(a, b)$ for all $(a, b) \in U$. (10 marks)